OCR
Oxford Cambridge and RSA

Monday 12 May 2025 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **(a)** The complex number z is such that $|z| = 7$ and $\arg(z) = 2.2$ radians.
Express z in cartesian form. **[3]**

(b) Use an algebraic method to determine the exact square roots of $1 + (4\sqrt{3})\mathrm{i}$. **[5]**

2 Two vectors, **a** and **b**, are given by $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 13 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 6 \\ p \end{pmatrix}$ where p is a constant.

(a) Find expressions in terms of p for each of the following.

- **a.b**
- $\mathbf{a} \times \mathbf{b}$

[3]

(b) Hence or otherwise find the value of p in each of the following cases.

- **a** and **b** are perpendicular
- **a** and **b** are parallel

[2]

3 The roots of the equation $2x^2 + 3x + 5 = 0$ are denoted by α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. **[2]**

(b) Using the answers to part **(a)** determine the value of each of the following.

- $\alpha^2 + \beta^2$
- $\frac{1}{\alpha} + \frac{1}{\beta}$

[4]

4 Two transformations, T_A and T_B, are represented by matrices **A** and **B** respectively.

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) (i) Describe the transformation T_A. **[1]**

(ii) Explain geometrically why $\mathbf{A}^{-1} = \mathbf{A}$. **[1]**

The matrix **B** is given by $\mathbf{B} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$.

(b) Describe the transformation T_B. **[2]**

The transformation T_C is equivalent to T_A followed by T_B.

(c) Determine the single matrix which represents T_C. **[2]**

5 The locus L is defined by $L = \{z : z \in \mathbb{C}, |z - (20 + 15\mathrm{i})| \leqslant 7\}$.

(a) On the Argand diagram in the Printed Answer Booklet, sketch and label L. **[2]**

(b) Determine the value of $z \in L$ for which the value of $|z|$ is smallest. Give your answer in cartesian form. **[3]**

(c) Determine the largest value of arg(z) for $z \in L$. **[3]**

6 The equations of two lines, l_1 and l_2, are $l_1 : \mathbf{r} = \begin{pmatrix} 16 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -27 \\ -19 \end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 10 \\ -10 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 10 \\ 10 \end{pmatrix}$.

(a) Show that l_1 and l_2 intersect at a single point, P, giving the coordinates of P. **[5]**

O is the origin of the coordinate system. The point Q lies on the line segment OP.

(b) Comment on the claim that the distance OQ is less than 100. **[2]**

7 Prove by induction that $n! > 20^n$ for all integers $n \geqslant 52$. **[5]**

8 Matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 5+a & 5 & 1 \\ 3 & 13+a & 6 \\ a-4 & -20 & -9 \end{pmatrix}$ where a is a constant and all entries of **A** are integers.

The transformation represented by **A** is applied to a shape of volume 8 units.
The image shape has volume 40 units and the orientation of the image is reversed.

Determine the image under **A** of the point (1, 2, 3). **[7]**

9 **(a)** Prove that if z is any complex number then $z(z^*) = |z|^2$. **[1]**

(b) Prove that if w and z are any complex numbers then $(wz)^* = w^* z^*$. **[2]**

The complex number v is defined by $v = (10+3\mathrm{i})(9+4\mathrm{i})$.

(c) **In this question you must show detailed reasoning.**

Express v in the form $a+b\mathrm{i}$ where a and b are real. **[2]**

(d) **In this question you must show detailed reasoning.**

By considering $v(v^*)$, write 10573 as the product of two prime factors. **[3]**

END OF QUESTION PAPER